The Non-Negotiables

33 SAT Math formulas you still have to know, sorted by the question that needs them.

Ronak & Rashmi

33 FORMULAS · 1 SHEET



THE NON-NEGOTIABLES FORMULAS

The whole sheet

Print pages 3 to 5. The number is the card that shows the formula inside a real question. ■ formula first, then Desmos

Lines, percents and rates

1 No solution or infinitely many ■

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow$ no solution

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$ infinitely many

2 Parallel and perpendicular lines ■

$ax + by = c$ has slope $-\frac{a}{b}$

parallel: $m_1 = m_2$

perpendicular: $m_1 m_2 = -1$

3 Shifting a gra ■

$y = f(a$

moves the graph

4 Percent multipliers ■

$p\%$ more: $\times \left(1 + \frac{p}{100}\right)$

$p\%$ less: $\times \left(1 - \frac{p}{100}\right)$

5 Working together ■

$\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$

add the rates, never the times

6 Squared unit density ■

1 ft

$1\text{ ft}^2 = 12^2\text{ in}^2$

mass = den

THE BASICS UNDERNEATH

+ **Slope.** $m = \frac{y_2 - y_1}{x_2 - x_1}$ · $y = mx + b$ · $y - y_1 = m(x - x_1)$
+ **Distance and midpoint.** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ · $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
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Quadratics, polynomials and exponents

7 The discriminant ■

$D = b^2 - 4ac$

$D > 0$ two solutions $D = 0$ one

$D < 0$ no real solution

8 Sum and product of the solutions ■

$ax^2 + bx + c = 0$

sum $= -\frac{b}{a}$ product $= \frac{c}{a}$

9 The vertex

x_{vertex}

= midpoint of two

$a(x - h)^2 + k$

10 What each form of a quadratic shows ■

$a(x - h)^2 + k$: vertex (h, k)

$a(x - p)(x - q)$: zeros p, q

$ax^2 + bx + c$: y-intercept c

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$(a + b)^2 =$

$(a - b)^2 =$

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$(x - a)$ is a factor $\iff p(a) = 0$

remainder on dividing by $(x - a) = p(a)$

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$a^m \cdot a^n = a^{m+n}$ $\frac{a^m}{a^n} =$

a^{m-n} $(a^m)^n = a^{mn}$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$ $a^{-n} = \frac{1}{a^n}$

$a^2 - b^2 = (a - b)(a + b)$

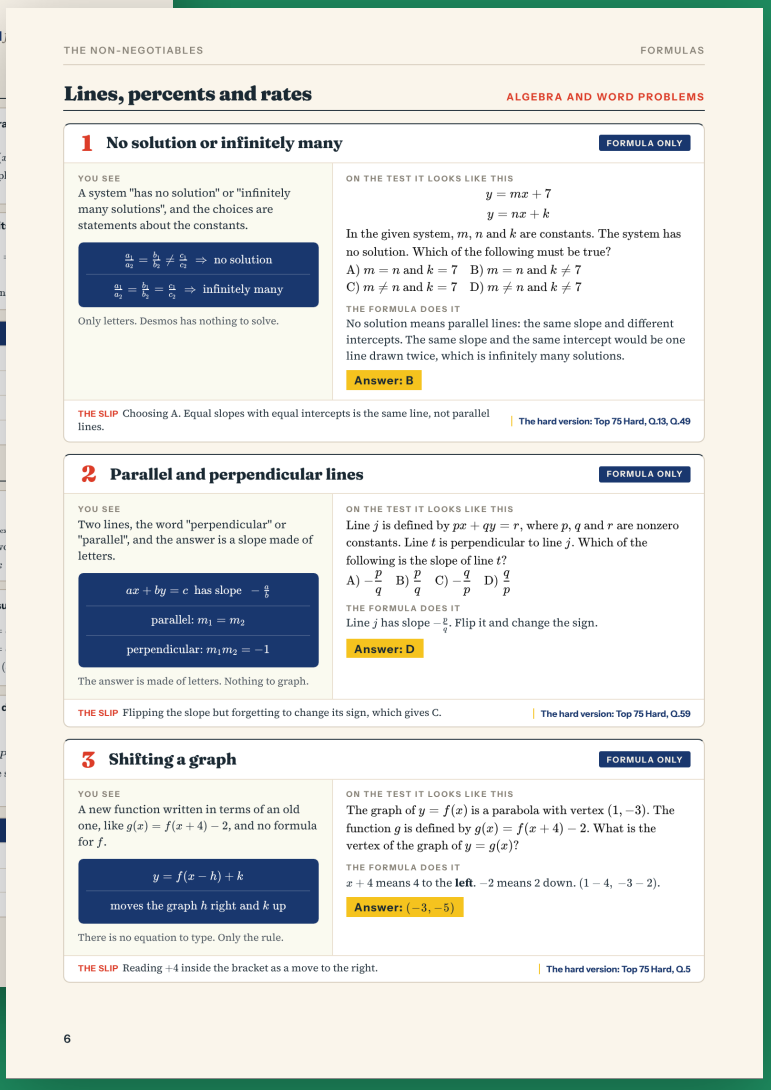
15 Growth and d step ■

$A = P$

$k =$ time for one

THE BASICS UNDERNEATH

+ **Quadratic formula.** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
+ **Linear or exponential?** Equal differences: linear. Equal ratios or a fixed percent: exponential.
+ **Zero and negative powers.** $a^0 = 1$ · $\frac{a^m}{a^n} = a^{m-n}$



THE NON-NEGOTIABLES FORMULAS

Lines, percents and rates

ALGEBRA AND WORD PROBLEMS

1 No solution or infinitely many

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YOU SEE

A system "has no solution" or "infinitely many solutions", and the choices are statements about the constants.

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Only letters. Desmos has nothing to solve.

ON THE TEST IT LOOKS LIKE THIS

$y = mx + 7$

$y = nx + k$

In the given system, m, n and k are constants. The system has no solution. Which of the following must be true?

A) $m = n$ and $k = 7$ B) $m = n$ and $k \neq 7$

C) $m \neq n$ and $k = 7$ D) $m \neq n$ and $k \neq 7$

THE FORMULA DOES IT

No solution means parallel lines: the same slope and different intercepts. The same slope and the same intercept would be one line drawn twice, which is infinitely many solutions.

Answer: B

THE SLIP Choosing A. Equal slopes with equal intercepts is the same line, not parallel lines.

The hard version: Top 75 Hard, Q.13, Q.49

2 Parallel and perpendicular lines

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Two lines, the word "perpendicular" or "parallel", and the answer is a slope made of letters.

$ax + by = c$ has slope $-\frac{a}{b}$

parallel: $m_1 = m_2$

perpendicular: $m_1 m_2 = -1$

The answer is made of letters. Nothing to graph.

ON THE TEST IT LOOKS LIKE THIS

Line j is defined by $px + qy = r$, where p, q and r are nonzero constants. Line t is perpendicular to line j. Which of the following is the slope of line t?

A) $-\frac{p}{q}$ B) $\frac{p}{q}$ C) $-\frac{q}{p}$ D) $\frac{q}{p}$

THE FORMULA DOES IT

Line j has slope $-\frac{p}{q}$. Flip it and change the sign.

Answer: D

THE SLIP Flipping the slope but forgetting to change its sign, which gives C.

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3 Shifting a graph

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A new function written in terms of an old one, like $g(x) = f(x + 4) - 2$, and no formula for f.

$y = f(x - h) + k$

moves the graph h right and k up

There is no equation to type. Only the rule.

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The graph of $y = f(x)$ is a parabola with vertex $(1, -3)$. The function g is defined by $g(x) = f(x + 4) - 2$. What is the vertex of the graph of $y = g(x)$?

THE FORMULA DOES IT

$x + 4$ means 4 to the **left**. -2 means 2 down. $(1 - 4,\ -3 - 2)$.

Answer: $(-3, -5)$

THE SLIP Reading $+4$ inside the bracket as a move to the right.

The hard version: Top 75 Hard, Q.5

READ THIS FIRST

You do not have a formula problem. You have a "which one, when" problem.

Almost 90% of our students ask us the same question. It usually comes right after they learn a few Desmos tricks:

> ***"In the exam, how do I know if this question is Desmos, a formula, or both? And how do I remember all of it?"***

If that is you, nothing is wrong with you. Most formula sheets are a list of 80 formulas in a random order. A list does not help when a question is in front of you and the clock is running.

So we built this one the other way round. We started from the questions that actually appear, and kept only the formulas that a calculator cannot replace or cannot beat. There are 33 of them. We call them the non-negotiables.

Every formula carries one of two labels

FORMULA ONLY

There is nothing to type. Letters in place of numbers, a figure, a rule, or a question about meaning. **29 formulas.**

FORMULA FIRST, THEN DESMOS

The formula turns the question into something a calculator can finish. **4 formulas.**

We tested every question in this book in Desmos, including the regression and slider tricks. If the calculator could reach the answer without the formula, we rewrote the question or moved that question type to the page called "What you can stop memorising". What is left truly needs the formula.

How to use it

1. **Start with pages 3 to 5.** That is the whole sheet. Print it and keep it in front of you.
2. **Then go card by card.** Read "You see" first. That line is the trigger you are training.
3. **Cover the right side** and try the question before you read our two lines.
4. **Read "The slip".** These are not traps. They are what a tired brain does at minute 30.

The formulas already printed on the test's own reference sheet, like the area of a circle or the volume of a cone, are not repeated here. You do not need to memorise those.

The whole sheet

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add the rates, never the times

6 Squared units, cubed units, density ■

$1 \text{ ft} = 12 \text{ in}$

$1 \text{ ft}^2 = 12^2 \text{ in}^2 \qquad 1 \text{ ft}^3 = 12^3 \text{ in}^3$

mass = density × volume

THE BASICS UNDERNEATH

- **Slope.** $m = \frac{y_2 - y_1}{x_2 - x_1}$ · $y = mx + b$ · $y - y_1 = m(x - x_1)$
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$x_{\text{vertex}} = -\frac{b}{2a}$

= midpoint of two equal-height points

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$a^{\frac{m}{n}} = \sqrt[n]{a^m} \qquad a^{-n} = \frac{1}{a^n}$

$a^2 - b^2 = (a - b)(a + b)$

15 Growth and decay with a time step ■

$A = P(1 \pm r)^{\frac{t}{k}}$

k = time for one step, in the unit of t

THE BASICS UNDERNEATH

- **Quadratic formula.** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- **Linear or exponential?** Equal differences: linear. Equal ratios or a fixed percent: exponential.
- **Zero and negative powers.** $a^0 = 1$ · $\frac{a^m}{a^n} = a^{m-n}$

The whole sheet, continued

■ *formula only* ■ *formula first, then Desmos*

Triangles, circles and solids

16 Angles of a triangle ■

the three angles add to $180°$

exterior angle = sum of the two far angles

17 Regular polygons ■

sum of interior angles $= (n-2)\cdot 180°$

each exterior angle $= \frac{360°}{n}$

18 The triangle inequality ■

$$|a-b| < c < a+b$$

between the difference and the sum

19 Similar figures: k, k², k³ ■

lengths $\times\, k$

areas $\times\, k^2$ volumes $\times\, k^3$

20 Equilateral and special right triangles ■

equilateral triangle, side s

height $= \frac{\sqrt{3}}{2}s$ area $= \frac{\sqrt{3}}{4}s^2$

21 Sine and cosine of the two acute angles ■

$\sin = \frac{\text{opp}}{\text{hyp}}$ $\cos = \frac{\text{adj}}{\text{hyp}}$ $\tan = \frac{\text{opp}}{\text{adj}}$

$\sin A = \cos B$ when $A + B = 90°$

22 The altitude to the hypotenuse ■

$$h = \frac{ab}{c}$$

all three triangles are similar

23 Arcs and sectors ■

arc $= \frac{\theta}{360}\cdot 2\pi r$ sector $= \frac{\theta}{360}\cdot \pi r^2$

radians: arc $= r\theta$ sector $= \frac{1}{2}r^2\theta$

π radians $= 180°$

24 Tangents and inscribed angles ■

tangent $\perp$ radius where they meet

inscribed angle $= \frac{1}{2}$ central angle

25 Circle in general form ■

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

center $(-g, -f)$ $r = \sqrt{g^2 + f^2 - c}$

26 Tangent line to a circle ■

$$m_{\text{radius}} = \frac{y_1 - k}{x_1 - h}$$

$$m_{\text{tangent}} = -\frac{1}{m_{\text{radius}}}$$

27 The right triangle inside a solid ■

cone: $\ell^2 = r^2 + h^2$

box diagonal: $d^2 = l^2 + w^2 + h^2$

THE BASICS UNDERNEATH

- **Circle, standard form.** $(x-h)^2 + (y-k)^2 = r^2$
- **Areas not on the reference sheet.** Trapezoid $\frac{1}{2}(b_1 + b_2)h$ · parallelogram bh
- **Parallel lines.** Angles are equal or add to $180°$. Acute + obtuse $= 180°$.
- **Triples worth knowing.** 3-4-5, 5-12-13, 8-15-17, 7-24-25 and their multiples.

The whole sheet, continued

■ *formula only* ■ *formula first, then Desmos*

Data, models and probability

28 Mean means sum ■

$$\text{sum} = \text{mean} \times \text{number of values}$$

29 What moves the mean, the median and the spread ■

an extreme value moves the mean

the median usually stays

standard deviation = spread

30 "Given that" picks the denominator ■

$$P(A \text{ given } B) = \frac{\text{in both } A \text{ and } B}{\text{in } B}$$

31 Margin of error ■

plausible = estimate ± margin of error

a sample speaks only for its own group

32 Linear or exponential ■

same amount each step: linear

same percent each step: exponential

33 What the number in a model means ■

$y = a + bx$ $y = a\,(b)^x$

a = the value when $x = 0$

b = change (or factor) per 1 unit of x

THE BASICS UNDERNEATH

+ **Probability.** $\frac{\text{outcomes you want}}{\text{all possible outcomes}}$
+ **Median.** The middle value once the data are in order. For n values it is position $\frac{n+1}{2}$.
+ **Range.** Greatest value minus least value.

ALREADY ON THE TEST'S OWN REFERENCE SHEET

✓ Area and circumference of a circle. Area of a rectangle and a triangle. The Pythagorean theorem. The 30-60-90 and 45-45-90 side ratios.

✓ Volume of a box, cylinder, sphere, cone and pyramid. A circle has 360 degrees or 2π radians. The angles of a triangle add to 180 degrees.

! They are given, but looking them up costs time. Know where they are, and know the two triangle ratios cold.

Is a formula you were told to memorise missing?

Check the page called "What you can stop memorising" near the end of this book. For many question types there is no formula to carry at all.

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