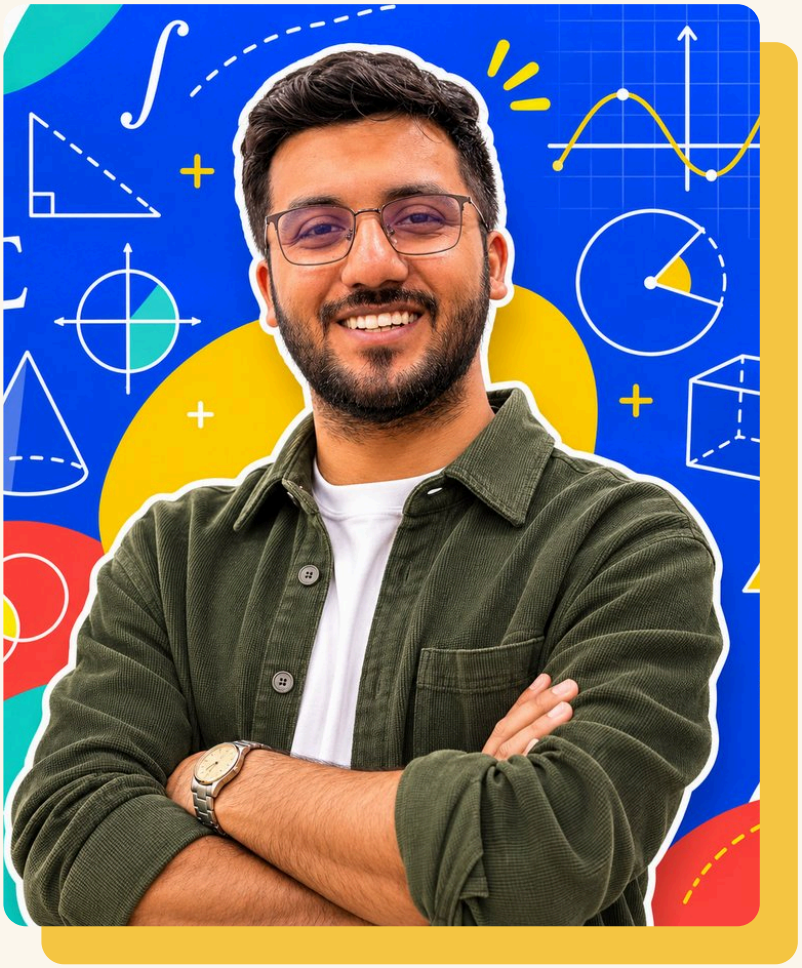
BEFORE THE 5 DECEMBER SAT

# Desmos can get you an 800. It can't guarantee it.



*Hey there, Math Hustlers.*

Algebra and Advanced Math, at Module 2 level. Fifteen questions, and one habit: decide before you solve.

TODAY

# What we cover in Session 1.

| # | PART | WHAT YOU GET |
|---|---|---|
| 1 | Four SATs, one pattern | What students saw at the end of Module 2 in August, September, October and November |
| 2 | The one rule of this bootcamp | What gets you to 780+ in Module 1, and what Module 2 adds |
| 3 | Warm-up | One Module 2 question to start the engine |
| 4 | Part 1 · Desmos wins | Questions 1 to 10: the right move turns minutes into seconds |
| 5 | Part 2 · Desmos lies, hand wins | Questions 11 to 15: where the calculator fails you, and the pen is faster |

THE BOOTCAMP

| SESSION | WHEN | TOPICS |
|---|---|---|
| **Session 1** | Today | Algebra and Advanced Math: systems, quadratics, constants, equivalent expressions |
| **Session 2** | Next | Geometry and Trigonometry, then Problem Solving and Data, percentages included |
| **Session 3** | Last | The theory you cannot miss, and your test-day plan for Module 2 |

HOW TO USE TODAY

Before you touch anything, decide: calculator or pen? Then solve. Write your answer before I explain, and keep a note of every question you got wrong and why.

WHAT THE LAST FOUR SATS SHOWED

# Four SATs, one pattern.

What students described after each test. Same shapes, new numbers, month after month.

## Aug 22
AUGUST

The last question was a percent chain or a median read from histograms. Parallel lines with a range of values appeared.

## Sep 12
SEPTEMBER

Letters everywhere. Students said the hard section was full of constants, and it ended on factoring with two letters.

## Oct 3
OCTOBER

"Not enough information" and "I only / II only" choices jumped. A factored cubic with a free constant closed the test.

## Nov 7
NOVEMBER

A table of f(x) plus a constant closed Module 2. Many grid-in answers were decimals, not whole numbers.

**1** **Module 2 turns hard late.**
Easy to medium until about question 15, then four to six slow questions. Time is the top complaint every month.

**2** **The hard questions are made of letters.**
Constants like k, a and b instead of numbers. Often the answer is an expression or a relationship.

**3** **Answers are not always whole numbers.**
5.5, 17/6, 31/3. If you hunt with a slider in steps of 1, you never land.

**4** **The question asks for something slightly different.**
The y-intercept of f, not of f + 19. $n^2/36$, not n. Read the last line twice.

THE ONE RULE OF THIS BOOTCAMP

# Calm. Accurate. Then fast.

MODULE 1
### Calm and accurate

- Zero careless mistakes is the target. 780+ starts here.
- Every question is doable. Speed comes from knowing the move, not from hurrying.
- Check the ask before you submit.

MODULE 2
### Calm, accurate, and on time

- The same calm and accuracy, on harder letters and conditions.
- Decide in seconds: calculator, pen, or both.
- Time management gets its own session later.

EVERY QUESTION TODAY GOES ONE OF TWO WAYS

### Desmos wins

The right move does in 30 seconds what algebra does in 3 minutes. The wrong move (dragging a slider) wastes the same 3 minutes.

### Desmos lies / hand wins

Desmos gives a confident wrong number, cannot handle the letters, or takes far too long. Here the pen is faster and safer.

WARM-UP · MODULE 2 LEVEL

DESMOS BIBLE · WARM-UP W03

Each solution of $x^2 + bx + c = 0$ is 3 more than a solution of $x^2 - 5x + 2 = 0$, where $b$ and $c$ are constants. What is the value of $c$?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

**Just for a good start.** We cover this idea properly later in the bootcamp. Try it now anyway.

STUCK IN SAT
Your Trusted Ally

# 01

PART 1 · QUESTIONS 1 TO 10

# Desmos wins.

*The right move, typed once. No sliders, no guessing.*

IN THIS PART

| | |
|---|---|
| **Q1** | Desmos Bible · N07 |
| **Q2** | Desmos Bible · N02 |
| **Q3** | Desmos Bible · N11 |
| **Q4** | Desmos Bible · V26-1 |
| **Q5** | Desmos Bible · V11-13 |
| **Q6** | Desmos Bible · V14-18 |
| **Q7** | Desmos Bible · V17-6 |
| **Q8** | Desmos Bible · Dec 5 update D5-02 |
| **Q9** | Top 75 Hard · Q68 |
| **Q10** | Desmos Bible · V09-11 |

**BEFORE EACH QUESTION**

Decide first: Desmos, pen, or both? Then solve, and write your answer before I explain.

**1** MODULE 2 LEVEL

DESMOS BIBLE · N07

For the quadratic function $f$, the table shows three values of $x$ and their corresponding values of $f(x) + 19$.

| $x$ | $f(x) + 19$ |
|---|---|
| 12 | $-987$ |
| 20 | $-211$ |
| 25 | 534 |

What is the $y$-coordinate of the $y$-intercept of the graph of $y = f(x)$ in the $xy$-plane?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT
Your Trusted Ally

2 MODULE 2 LEVEL

DESMOS BIBLE · N02

In the $xy$-plane, line $\ell$ has slope $m$, where $m > 1$, and passes through the point $(5, -3)$. Line $k$ is perpendicular to line $\ell$ and also passes through the point $(5, -3)$. Which of the following points lies on line $k$?

(A) $\left(6,\ -3 + \frac{1}{m}\right)$

(B) $(6,\ -3 - m)$

(C) $(5 - m,\ -4)$

(D) $(5 + m,\ -4)$

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT Your Trusted Ally

**3** MODULE 2 LEVEL

DESMOS BIBLE · N11

$$3x^2 - 18x + 13 = 0$$

One solution to the given equation can be written as $3 + \frac{\sqrt{k}}{3}$, where $k$ is a constant. What is the value of $k$?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT
Your Trusted Ally

4 MODULE 2 LEVEL

DESMOS BIBLE · V26-1

$$\frac{4}{15}by + \frac{2}{5} = -\frac{4}{15}ax$$
$$\frac{1}{4}x + \frac{1}{15}y = \frac{a}{3} + \frac{3}{4}x$$

In the given system of equations, $a$ and $b$ are constants and $a > 0$. The system has infinitely many solutions. What is the value of $a - b$?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT
Your Trusted Ally

**5** MODULE 2 LEVEL

DESMOS BIBLE · V11-13

$$(k+1)x - 18y = 4k$$

$$6x - 5 = (2k-4)y$$

In the given system of equations, $k$ is a constant and $k > 0$. If the system has no solution, what is the value of $k$?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT
Your Trusted Ally

**6** MODULE 2 LEVEL

DESMOS BIBLE · V14-18

The function $f$ is defined by $f(x) = 640(3.2)^{\frac{x}{4}}$. It can be rewritten as $f(x) = 640\left(1 + \frac{k}{100}\right)^{x}$, where $k$ is a constant. What is the value of $k$, rounded to the nearest tenth?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT
Your Trusted Ally

7 MODULE 2 LEVEL

DESMOS BIBLE · V17-6

A quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. The model indicates the object hits the ground 14 seconds after being launched; the object reaches its maximum height of 1,024 feet above the ground 6 seconds after being launched. Based on the model, at what time after the object is launched, in seconds, does the object first reach a height of 924 feet?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT
Your Trusted Ally

**8** MODULE 2 LEVEL

DESMOS BIBLE · DEC 5 UPDATE D5-02

$$\frac{ax}{x-3} - \frac{4}{2x+b} = \frac{6x^2+11x+4b-8}{(x-3)(2x+b)}$$

The equation above is true for all values of $x$ where $x \neq 3$ and $x \neq -\frac{b}{2}$, and $a$ and $b$ are constants. What is the value of $a+b$?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT
Your Trusted Ally

**9** MODULE 2 LEVEL

TOP 75 HARD · Q68

$$y = x^2 - 5x + 10$$

$$y = k - 2x$$

In the given system of equations, $k$ is a constant. The graphs of the equations in the $xy$-plane intersect at exactly one point. What is the value of $k$?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT
Your Trusted Ally

**10** MODULE 2 LEVEL

DESMOS BIBLE · V09-11

$$5(2p-k)(x+3)-2(-3-x)^2=5(4p-k)(x+3)$$

In the given equation, $p$ and $k$ are constants, and the equation has exactly two distinct real solutions. If the sum of the solutions is $-36$, what is the value of $p$?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT
Your Trusted Ally

# 02

PART 2 · QUESTIONS 11 TO 15

# Desmos lies. Hand wins.

*A confident wrong number is worse than no number.*

IN THIS PART

| | |
|---|---|
| **Q11** | Desmos Bible · V15-6 |
| **Q12** | Top 75 Hard · Q42 |
| **Q13** | Desmos Bible · V27-4 |
| **Q14** | Desmos Bible · V09-18 |
| **Q15** | Top 75 Hard · Q8 |

**BEFORE EACH QUESTION**

Ask: can Desmos even see this? Letters only, a hidden zero, a huge power? Then the pen goes first.

**11** MODULE 2 LEVEL

DESMOS BIBLE · V15-6

The function $h$ is defined by

$$h(x) = -\sqrt{x^3 - bx + c},$$

where $b$ and $c$ are constants. In the $xy$-plane, the graph of $y = h(x)$ contains the points $(4, 0)$ and $\left(0, -\sqrt{468}\right)$. If $h(k) = 0$, what is the least possible value of $k$?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT Your Trusted Ally

**12** MODULE 2 LEVEL

TOP 75 HARD · Q42

$$g(x) = 8(x - d)(x - e)(x - f)$$

The function $g$ is defined by the given equation, where $d$, $e$, and $f$ are distinct constants. When $d < x < f$, the value of $g(x)$ is positive. The graph of $y = g(x)$ in the $xy$-plane contains the point $(p, q)$, where $p$ and $q$ are constants. If $q = 7$, which of the following could be true?

I. $e < p$

II. $p < d$

III. $f < p < e$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

**13** MODULE 2 LEVEL

DESMOS BIBLE · V27-4

Which of the following expressions has $4x - 3b$ as a factor, where $b$ is a positive integer constant?

(A) $12x^2 + 12x - 9b$

(B) $12x^2 - 15x - 9b$

(C) $12x^2 + 24x - 9b$

(D) $12x^2 - 225x - 9b$

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT Your Trusted Ally

**14** MODULE 2 LEVEL

DESMOS BIBLE · V09-18

The quadratic function $h$ is defined by $h(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are constants and $a \neq 0$. It is given that $h(-5) > 0$, $h(3) < 0$, and $h(21) > 0$. Which of the following could NOT be the $x$-coordinate of the vertex of the graph of $y = h(x)$?

(A) $-2$

(B) $2$

(C) $7$

(D) $11$

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT Your Trusted Ally

**15** MODULE 2 LEVEL

TOP 75 HARD · Q8

$$f(x) = 11x^{10} + 2x^8$$

$$g(x) = -14x^7 + 9x^5$$

The polynomial $p(x)$ is the product of the given polynomials $f(x)$ and $g(x)$. What is the coefficient of $x^{15}$ in $p(x)$?

Your answer

Decide first: ☐ Desmos ☐ Hand ☐ Both Time taken _____

STUCK IN SAT Your Trusted Ally

# 03

PART 3 · ANSWERS AND SOLUTIONS

# Check yourself.

*Answer key, the systems card, and a full solution for every question.*

## HOW TO USE THIS PART

**TRY FIRST, THEN CHECK**

Redo every question you got wrong in class without looking. Then check the answer key, and read the full solution only for the ones you still miss.

- Every solution shows the trap most students fall into, the Desmos route (the exact lines to type), the by-hand route, and which one wins.
- Desmos lines are typed exactly as shown. ~ is the tilde key, and $x_1$ is typed x_1.

ANSWER KEY

# All 16 answers.

Mark yourself first. Then read the solution for every one you missed.

| # | ANSWER | GROUP | CODE |
|---|---|---|---|
| ★ | $c = 26$ | Warm-up | Desmos Bible · Warm-up W03 |
| Q1 | $-1210$ | Desmos wins | Desmos Bible · N07 |
| Q2 | **D** | Desmos wins | Desmos Bible · N02 |
| Q3 | $k = 42$ | Desmos wins | Desmos Bible · N11 |
| Q4 | $a - b = \frac{17}{10}$ | Desmos wins | Desmos Bible · V26-1 |
| Q5 | $k = 8$ | Desmos wins | Desmos Bible · V11-13 |
| Q6 | $k = 33.7$ | Desmos wins | Desmos Bible · V14-18 |
| Q7 | $3.5$ **seconds** | Desmos wins | Desmos Bible · V17-6 |
| Q8 | $a + b = 8$ | Desmos wins | Desmos Bible · Dec 5 update D5-02 |
| Q9 | $k = 7.75$ | Desmos wins | Top 75 Hard · Q68 |
| Q10 | $p = 6$ | Desmos wins | Desmos Bible · V09-11 |
| Q11 | $-13$ | Desmos lies | Desmos Bible · V15-6 |
| Q12 | **A (I only)** | Hand wins | Top 75 Hard · Q42 |
| Q13 | **B** ($b = 3$) | Hand wins | Desmos Bible · V27-4 |
| Q14 | **A** ($-2$) | Hand wins | Desmos Bible · V09-18 |
| Q15 | $71$ | Hand wins | Top 75 Hard · Q8 |

CARD · Q4 AND Q5

# Same line or parallel?

Two systems questions, two Desmos lines. Keep this card next to Desmos.

Q4 · INFINITELY MANY

## Same line

1. Type $x_1 = [1, 2, 3]$ and $y_1 = [1, -1, 2]$. Any three points work, as long as they are not on one straight line (so never the same list twice).
2. Type (left − right of eq 1) ~ $t$·(left − right of eq 2). Numbers IN.
3. Add the question's condition, like $\{a > 0\}$.
4. Read the letters.

Q5 · NO SOLUTION

## Parallel

1. Do NOT give $x_1$ or $y_1$ values. Desmos finds them.
2. Bring every x and y term to the left. Delete the numbers.
3. Type [x, y part of eq 1, x, y part of eq 2] ~ $[0, 0]\{x_1 > 0\}$.
4. Read $k$, then check the numbers are NOT in the same ratio. If they are, it is the same line, not parallel.

BY HAND, ALWAYS WORKS

| CASE | RULE FOR $A_1X + B_1Y = C_1$ AND $A_2X + B_2Y = C_2$ |
|---|---|
| **Infinitely many** | $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$ |
| **No solution** | $\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$ |

WATCH OUT

| | |
|---|---|
| **Numbers typed in, no-solution line** | Desmos gives a confident wrong decimal (like $-8.667$, a wrong choice). Numbers OUT for parallel. |
| **Letter only in the number term** | The parallel line cannot find it (deleting numbers deletes the letter). Use the ratio rule by hand. |
| **No number at all** | Both lines pass through (0, 0), so they always meet. Never "no solution". |

SOLUTION · WARM-UP

DESMOS BIBLE · WARM-UP W03

Each solution of $x^2 + bx + c = 0$ is 3 more than a solution of $x^2 - 5x + 2 = 0$, where $b$ and $c$ are constants. What is the value of $c$?

**ANSWER** $c = 26$

**THE TRAP**

Try to find the roots with the quadratic formula, get $\frac{5 \pm \sqrt{17}}{2}$, add 3, then expand $(x - r_1)(x - r_2)$ by hand. Slow, and the radicals cause sign slips.

**DESMOS ROUTE**

LINE 1 $f(x) = x^2 - 5x + 2$

LINE 2 $x_1 = [1, 2, 3]$

LINE 3 $x_1^2 + bx_1 + c \sim f(x_1 - 3)$
*every root 3 more = the graph moves 3 right*

→ $b = -11,\ c = 26$ (exact).

Another way: click the two roots of $x^2 - 5x + 2$ (0.438 and 4.562), add 3 to each, then $x_1^2 + bx_1 + c \sim (x_1 - 3.438)(x_1 - 7.562)$. You get $c \approx 26$. Typing $f(x_1 - 3)$ skips the decimals and gives exactly 26.

**BY HAND**

1. New roots are $r_1 + 3$ and $r_2 + 3$.
2. $c$ = product of new roots $= (r_1 + 3)(r_2 + 3) = r_1 r_2 + 3(r_1 + r_2) + 9$.
3. From $x^2 - 5x + 2$: $r_1 + r_2 = 5$, $r_1 r_2 = 2$. So $c = 2 + 15 + 9 = 26$.

**Verdict:** Both are fast. Desmos about 30 s; by hand about 30 s if they know sum and product.

# 1 SOLUTION · DESMOS WINS

DESMOS BIBLE · N07

For the quadratic function $f$, the table shows three values of $x$ and their corresponding values of $f(x) + 19$.

| $x$ | $f(x) + 19$ |
|---|---|
| 12 | $-987$ |
| 20 | $-211$ |
| 25 | 534 |

What is the $y$-coordinate of the $y$-intercept of the graph of $y = f(x)$ in the $xy$-plane?

**ANSWER** $-1210$

**THE TRAP**

Run the regression on the table, read the y-intercept $-1191$ and stop. That is the y-intercept of $f(x) + 19$. Others add 19 and get $-1172$.

**DESMOS ROUTE**

**LINE 1** $x_1 = [12, 20, 25]$

**LINE 2** $y_1 = [-987, -211, 534]$

**LINE 3** $y_1 \sim ax_1^2 + bx_1 + c + 19$
*put the +19 inside the model*

$\rightarrow c = -1210$. Because the 19 is inside the model, $c$ is already the y-intercept of $f$ itself.

Or regress $y_1 \sim ax_1^2 + bx_1 + d$ normally ($d = -1191$) and then subtract 19.

**BY HAND**

1. Write $f(x) = ax^2 + bx + c$ and use three equations: $144a + 12b + c + 19 = -987$, $400a + 20b + c + 19 = -211$, $625a + 25b + c + 19 = 534$.
2. Subtract pairs to remove $c$, solve for $a$ and $b$, then $c$. About 3 to 4 minutes and easy to slip.

**Verdict:** Desmos wins clearly: about 40 s against 3 to 4 minutes.

## 2 SOLUTION · DESMOS WINS

DESMOS BIBLE · N02

In the $xy$-plane, line $\ell$ has slope $m$, where $m > 1$, and passes through the point $(5, -3)$. Line $k$ is perpendicular to line $\ell$ and also passes through the point $(5, -3)$. Which of the following points lies on line $k$?

A $\left(6,\ -3 + \dfrac{1}{m}\right)$

B $(6,\ -3 - m)$

C $(5 - m,\ -4)$

D $(5 + m,\ -4)$

**ANSWER D**

**THE TRAP**

Use slope $\dfrac{1}{m}$ (A) or $-m$ (B). Or look only for a choice with $x = 6$; the true point $\left(6, -3 - \frac{1}{m}\right)$ is not offered.

**DESMOS ROUTE**

**LINE 1** $m = 2$
*any legal value, m > 1*

**LINE 2** $x_1 = [5]$

**LINE 3** $y_1 = [-3]$
*the one point we know*

**LINE 4** $y_1 \sim -\frac{1}{m}x_1 + c$
*slope is known, so only c is unknown*

**LINE 5** $X = [6, 6, 5 - m, 5 + m]$

**LINE 6** $Y = \left[-3 + \frac{1}{m}, -3 - m, -4, -4\right]$

**LINE 7** $Y - \left(-\frac{1}{m}X + c\right)$
*0 = the choice is on line k*

→ $c = -0.5$, then $[1,\ -1.5,\ -2,\ 0]$: only D gives 0. Change $m$ to 3: D is still 0.

Why this works: you know one point and the slope, so only $c$ is unknown. No point-slope formula needed. Any legal $m$ gives the same choice, so try $m = 3$ to be sure.

**BY HAND**

1. Line $k$ has slope $-\dfrac{1}{m}$ through $(5, -3)$.
2. Move $m$ to the right and 1 down: $(5 + m,\ -3 - 1) = (5 + m, -4)$. That is D.

**Verdict:** Desmos about 45 s and works for every student. Hand 15 s for students who see 'run m, rise −1'.

## 3 SOLUTION · DESMOS WINS

DESMOS BIBLE · N11

$$3x^2 - 18x + 13 = 0$$

One solution to the given equation can be written as $3 + \dfrac{\sqrt{k}}{3}$, where $k$ is a constant. What is the value of $k$?

**ANSWER** $k = 42$

**THE TRAP**

Complete the square to $(x - 3)^2 = \dfrac{14}{3}$ and answer 14. Or click the root in Desmos and try to reverse the decimal.

**DESMOS ROUTE**

LINE 1 $x_1 = 3 + \frac{\sqrt{k}}{3}$
*the form from the question*

LINE 2 $3x_1^2 - 18x_1 + 13 \sim 0$

$\rightarrow k = 42.$

**BY HAND**

1. $x = \dfrac{18 \pm \sqrt{324 - 156}}{6} = 3 \pm \dfrac{\sqrt{168}}{6}.$
2. $\dfrac{\sqrt{168}}{6} = \dfrac{\sqrt{168/4}}{3} = \dfrac{\sqrt{42}}{3}$, so $k = 42.$

**Verdict:** Desmos about 30 s. Hand about 60 s with the trap at 14.

## 4 SOLUTION · DESMOS WINS

DESMOS BIBLE · V26-1

$$\frac{4}{15}by + \frac{2}{5} = -\frac{4}{15}ax$$
$$\frac{1}{4}x + \frac{1}{15}y = \frac{a}{3} + \frac{3}{4}x$$

In the given system of equations, $a$ and $b$ are constants and $a > 0$. The system has infinitely many solutions. What is the value of $a - b$?

**ANSWER** $a - b = \frac{17}{10}$

**THE TRAP**

Leave the $\frac{3}{4}x$ on the right, match only the x and y parts and forget the numbers, or try two sliders at once.

### DESMOS ROUTE

**LINE 1** $x_1 = [1, 2, 3]$
*any x values*

**LINE 2** $y_1 = [1, -1, 2]$
*any y values, NOT the same as $x_1$*

**LINE 3** $\frac{4}{15}by_1 + \frac{2}{5} + \frac{4}{15}ax_1 \sim t\left(\frac{1}{4}x_1 + \frac{1}{15}y_1 - \frac{a}{3} - \frac{3}{4}x_1\right)\{a > 0\}$
*(left − right of eq 1) ~ t × (left − right of eq 2), numbers IN*

**LINE 4** $a - b$

→ $a = 1.5,\ b = -0.2$, so $a - b = 1.7 = \frac{17}{10}$.

Why $t$: same line means equation 1 is equation 2 multiplied by some number. $t$ is that number, and Desmos finds it. Why lists for $x_1$ and $y_1$: the two sides must match at every point, not just one. No moving terms: type each equation as it is, left minus right, numbers included.

### BY HAND

1. Ratios: $\frac{\frac{4}{15}a}{-\frac{1}{2}} = \frac{\frac{4}{15}b}{\frac{1}{15}} = \frac{-\frac{2}{5}}{\frac{a}{3}}$.
2. Middle = right: $4b = -\frac{6}{5a}$; left = middle: $-\frac{8a}{15} = 4b$.
3. $\frac{8a}{15} = \frac{6}{5a}, a^2 = \frac{9}{4}, a = \frac{3}{2}, b = -\frac{1}{5}$. $a - b = \frac{17}{10}$.

**Verdict:** Desmos wins: about 60 s against 3+ minutes of fractions.

**5** SOLUTION · DESMOS WINS

DESMOS BIBLE · V11-13

$$(k+1)x - 18y = 4k$$
$$6x - 5 = (2k-4)y$$

In the given system of equations, $k$ is a constant and $k > 0$. If the system has no solution, what is the value of $k$?

**ANSWER** $k = 8$

**THE TRAP**

Leave $(2k-4)y$ on the right and get the sign wrong, or solve the ratio, get $k^2 - k - 56 = 0$ and answer $-7$. Others keep the numbers $4k$ and $5$ in the regression.

**DESMOS ROUTE**

LINE 1 $[(k+1)x_1 - 18y_1,\ 6x_1 - (2k-4)y_1] \sim [0,0]\ \{x_1 > 0\}$

*x and y terms only (move every x, y term to the left, delete the numbers)*

$\rightarrow k = 8$. Check: $k = 8$ is positive ✓, and the numbers 32 and 5 are not in the same ratio as 9 and 6, so parallel, not the same line.

Why $\{x_1 > 0\}$: without it Desmos can make $x_1 = y_1 = 0$, which fits $[0,0]$ for any $k$. The restriction blocks that. Put it on $x_1$, not on $k$, then check $k$ against the question's own condition ($k > 0$).

**BY HAND**

1. Line up: $(k+1)x - 18y = 4k$ and $6x - (2k-4)y = 5$.
2. Parallel: $\dfrac{k+1}{6} = \dfrac{-18}{-(2k-4)} = \dfrac{9}{k-2}$.
3. $(k+1)(k-2) = 54 \rightarrow k^2 - k - 56 = 0 \rightarrow k = 8$ or $-7$. $k > 0 \rightarrow 8$.

**Verdict:** Desmos wins: one line, about 30 s. Hand about 60 s with a quadratic and a sign trap.

# 6 SOLUTION · DESMOS WINS

DESMOS BIBLE · V14-18

The function $f$ is defined by $f(x) = 640(3.2)^{\frac{x}{4}}$. It can be rewritten as $f(x) = 640\left(1 + \frac{k}{100}\right)^{x}$, where $k$ is a constant. What is the value of $k$, rounded to the nearest tenth?

**ANSWER** $k = 33.7$

**THE TRAP**

Share the growth evenly: 220% over 4 steps, so 55% per step (wrong; growth compounds). Others answer 220 or 133.7.

**DESMOS ROUTE**

LINE 1 $x_1 = [1, 2, 3]$

LINE 2 $640 \cdot 3.2^{\frac{x_1}{4}} \sim 640\left(1 + \frac{k}{100}\right)^{x_1}$
*replace x with $x_1$, = with ~*

→ $k = 33.748$, so $33.7$.

**BY HAND**

1. $3.2^{x/4} = \left(3.2^{1/4}\right)^{x}$.
2. $3.2^{1/4} = 1.3375$, so $1 + \frac{k}{100} = 1.3375$ and $k = 33.7$.

**Verdict:** Both fast (about 30 s). Desmos is the safe one for students who do not see the exponent rule.

7 SOLUTION · DESMOS WINS

DESMOS BIBLE · V17-6

A quadratic function models the height, in feet, of an object above the ground in terms of the time, in seconds, after the object is launched off an elevated surface. The model indicates the object hits the ground $14$ seconds after being launched; the object reaches its maximum height of $1{,}024$ feet above the ground $6$ seconds after being launched. Based on the model, at what time after the object is launched, in seconds, does the object first reach a height of $924$ feet?

**ANSWER** $3.5$ **seconds**

**THE TRAP**

Give 8.5 (the second time at 924 ft, on the way down), or try standard form $ax^2 + bx + c$ with only two points.

**DESMOS ROUTE**

**LINE 1** $h(x) = a(x-6)^2 + 1024$
*vertex form from the maximum*

**LINE 2** $h(14) \sim 0$
*lands at 14 s*

**LINE 3** $y = 924$
*click the two crossings*

→ $a = -16$. Crossings $(3.5,\ 924)$ and $(8.5,\ 924)$. First = 3.5.

**BY HAND**

1. $h(14) = 0$: $64a + 1024 = 0$, $a = -16$.
2. $-16(x-6)^2 + 1024 = 924$, $(x-6)^2 = 6.25$, $x = 6 \pm 2.5$.
3. First time: $3.5$.

**Verdict:** Both about 45 s. The marks are lost in the reading ('first'), not the maths.

# 8 SOLUTION · DESMOS WINS

DESMOS BIBLE · DEC 5 UPDATE D5-02

$$\frac{ax}{x-3} - \frac{4}{2x+b} = \frac{6x^2+11x+4b-8}{(x-3)(2x+b)}$$

The equation above is true for all values of $x$ where $x \neq 3$ and $x \neq -\frac{b}{2}$, and $a$ and $b$ are constants. What is the value of $a+b$?

**ANSWER** $a+b=8$

**THE TRAP**

Type the fractions straight into a regression (Desmos says it can't find a region where the model works), or slip the sign: $-4(x-3) = -4x-12$, then the constants give $b=-1$.

## DESMOS ROUTE

**LINE 1** $x_1 = [1, 2, 3]$

**LINE 2** $ax_1(2x_1+b) - 4(x_1-3) \sim 6x_1^2 + 11x_1 + 4b - 8$

*same denominators → compare the tops only*

**LINE 3** $a+b$

→ $a=3,\ b=5$, so $a+b=8$.

The trap: typing the fractions straight into a regression gives an error ('couldn't find any region where this model is defined'), or a huge useless number. Same denominators on both sides, so compare the tops only.

## BY HAND

1. Top of the left side: $ax(2x+b) - 4(x-3) = 2ax^2 + (ab-4)x + 12$.
2. Match: $2a=6,\ ab-4=11,\ 12=4b-8$.
3. $a=3, b=5, a+b=8$.

**Verdict:** Desmos wins once the denominators are gone: about 40 s. Hand about 90 s.

# 9 SOLUTION · DESMOS WINS

TOP 75 HARD · Q68

$$y = x^2 - 5x + 10$$
$$y = k - 2x$$

In the given system of equations, $k$ is a constant. The graphs of the equations in the $xy$-plane intersect at exactly one point. What is the value of $k$?

**ANSWER** $k = 7.75$

**THE TRAP**

9 out of 10: graph both, put a slider on $k$, drag. Steps of 1 or 0.1 look tangent at 7.7 or 7.8; zoom in and it is 0 or 2 points.

## DESMOS ROUTE

**LINE 1** $x^2 - 5x + 10 = y - 2x$
*replace k with y, graph it, click the vertex: y = k touches the curve once only there*

→ Vertex $(1.5,\ 7.75)$, so $k = 7.75$.

Why it works: the system becomes $k = x^2 - 3x + 10$. Every horizontal line $y = k$ cuts this parabola twice, except the one through the vertex, which touches it once. Replace the constant with $y$, graph, click the vertex. No slider needed.

## BY HAND

1. $x^2 - 5x + 10 = k - 2x \rightarrow x^2 - 3x + (10 - k) = 0$.
2. One solution: $9 - 4(10 - k) = 0$, $k = \frac{31}{4} = 7.75$.

**Verdict:** Desmos wins: about 20 s (k as y, click vertex). Hand about 30 s. Slider: minutes, and still wrong.

# 10 SOLUTION · DESMOS WINS

DESMOS BIBLE · V09-11

$$5(2p-k)(x+3)-2(-3-x)^2=5(4p-k)(x+3)$$

In the given equation, $p$ and $k$ are constants, and the equation has exactly two distinct real solutions. If the sum of the solutions is $-36$, what is the value of $p$?

**ANSWER** $p=6$

**THE TRAP**

Divide both sides by $(x+3)$, lose $x=-3$, and get $p=6.6$. Others read $(-3-x)^2$ as $-(x+3)^2$.

## DESMOS ROUTE

**LINE 1** $k=1$
*k cancels, so any value works*

**LINE 2** $F(x)=5(2p-k)(x+3)-2(-3-x)^2-5(4p-k)(x+3)$
*left minus right*

**LINE 3** $[F(u),F(-36-u)]\sim[0,0]\,\{u>-18\}$
*two roots that add to −36, kept apart*

→ $p=6$ (roots $-3$ and $-33$).

How to pick the condition: the two roots add to $-36$, so they sit on either side of the middle, $-18$. If both were $-18$ there would be only one solution, but the question says two distinct. So tell Desmos one root is bigger than the middle: $\{u>-18\}$. Rule: $\{u>\text{sum}\div 2\}$.

## BY HAND

1. $(-3-x)^2=(x+3)^2$. Take out $(x+3)$: $(x+3)\,[5(2p-k)-2(x+3)-5(4p-k)]=0$.
2. Bracket: $-10p-2x-6=0$, so $x=-5p-3$. Other root: $x=-3$.
3. Sum: $-3+(-5p-3)=-36$, $p=6$. Note $k$ disappears.

**Verdict:** Desmos wins: about 40 s, no insight needed. Hand (factor out $x+3$) about 45 s.

# 11 SOLUTION · DESMOS LIES

DESMOS BIBLE · V15-6

The function $h$ is defined by

$$h(x) = -\sqrt{x^3 - bx + c},$$

where $b$ and $c$ are constants. In the $xy$-plane, the graph of $y = h(x)$ contains the points $(4, 0)$ and $\left(0, -\sqrt{468}\right)$. If $h(k) = 0$, what is the least possible value of $k$?

**ANSWER** $-13$

**THE TRAP**

Answer 4 (the zero already given), or 9 (the other zero inside the default window). Or regress both points at once and get bad values.

**DESMOS ROUTE**

**LINE 1** $c = 468$
*from (0, −√468): h(0) = −√c*

**LINE 2** $4^3 - 4b + c \sim 0$
*the inside is 0 at x = 4*

**LINE 3** $h(x) = -\sqrt{x^3 - 133x + 468}$
*then zoom out*

→ $b = 133$. Zoomed out, the graph touches $y = 0$ at $x = -13$, 4 and 9. Least: $-13$.

Why not one regression: with a square root, the fit is poor ($R^2 \neq 1$) and the numbers it gives can't be trusted. Get $c$ by hand first, then let Desmos do the rest.

**BY HAND**

1. $h(0) = -\sqrt{c} = -\sqrt{468}$, so $c = 468$. $h(4) = 0$: $64 - 4b + 468 = 0$, $b = 133$.
2. $x^3 - 133x + 468 = (x - 4)(x^2 + 4x - 117) = (x - 4)(x + 13)(x - 9)$.
3. Least zero: $-13$.

**Verdict:** Both: one fact by hand ($c = 468$), then Desmos. About 60 s.

# 12 SOLUTION · HAND WINS

TOP 75 HARD · Q42

$$g(x) = 8(x-d)(x-e)(x-f)$$

The function $g$ is defined by the given equation, where $d$, $e$, and $f$ are distinct constants. When $d < x < f$, the value of $g(x)$ is positive. The graph of $y = g(x)$ in the $xy$-plane contains the point $(p, q)$, where $p$ and $q$ are constants. If $q = 7$, which of the following could be true?

I. $e < p$
II. $p < d$
III. $f < p < e$

**A** I only
**B** II only
**C** I and II only
**D** I and III only

**ANSWER** **A (I only)**

**THE TRAP**
Assume $d < e < f$ (alphabetical order) and pick D. Or try to graph with letters and freeze.

**DESMOS ROUTE**

**LINE 1** $g(x) = 8(x-1)(x-3)(x-2)$
*legal values: d = 1, f = 2, e = 3*

**LINE 2** $y = 7$
*see where g = 7*

→ With these values $g = 7$ at $x \approx 3.3$, which is beyond $e = 3$, so I can be true. Left of 1 and between 2 and 3 the graph is negative, so II ($p < d$) and III ($f < p < e$) cannot be.

The key is choosing the order $d < f < e$ correctly first. That choice is the hand thinking; Desmos only confirms it.

**BY HAND**

1. Positive cubic signs from the left: $-, +, -, +$.
2. $g > 0$ on $d < x < f$ means $d$ and $f$ are neighbours: $d < f < e$.
3. $g(p) = 7 > 0$: $p$ is in $(d, f)$ or $p > e$. Only I.

**Verdict:** Hand wins (about 40 s). Desmos only after you choose legal values.

# 13 SOLUTION · HAND WINS

DESMOS BIBLE · V27-4

Which of the following expressions has $4x - 3b$ as a factor, where $b$ is a positive integer constant?

A $12x^2 + 12x - 9b$

B $12x^2 - 15x - 9b$

C $12x^2 + 24x - 9b$

D $12x^2 - 225x - 9b$

**ANSWER** **B** ( $b = 3$ )

**THE TRAP**

Graph each choice with a slider on $b$ and hunt for a common zero, or pick A because $b = 0$ 'works' (but $b$ must be a positive integer).

## DESMOS ROUTE

**LINE 1** $C = [12, -15, 24, -225]$
*the x-coefficients*

**LINE 2** $y = 12\left(\frac{3x}{4}\right)^2 + C \cdot \frac{3x}{4} - 9x$
*graph b as x: put x = 3b/4 in each*

→ Positive integer x-intercepts: only the $-15$ curve, at $x = 3$. So B, $b = 3$. (A gives 0, C gives $-\frac{4}{3}$, D gives $\frac{79}{3}$.)

## BY HAND

1. $4x - 3b$ is a factor, so $x = \dfrac{3b}{4}$ is a zero.
2. $12 \cdot \dfrac{9b^2}{16} + c \cdot \dfrac{3b}{4} - 9b = 0 \to 27b + 3c - 36 = 0 \to b = \dfrac{12 - c}{9}$.
3. $c = -15$: $b = 3$ ✓. Others: 0, $-\frac{4}{3}$, $\frac{79}{3}$.

**Verdict:** Hand about 60 s. Desmos (constant as x) about 60 s. Graphing the four choices directly does not work.

## 14 SOLUTION · HAND WINS

DESMOS BIBLE · V09-18

The quadratic function $h$ is defined by $h(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are constants and $a \neq 0$. It is given that $h(-5) > 0$, $h(3) < 0$, and $h(21) > 0$. Which of the following could NOT be the $x$-coordinate of the vertex of the graph of $y = h(x)$?

A $-2$
B $2$
C $7$
D $11$

**ANSWER A ( $-2$ )**

**THE TRAP**

Pick D (11) because it is farthest from 3. But the allowed range is lopsided.

### DESMOS ROUTE

**LINE 1** $-5 < x < 3\,\{3 < y < 21\}$
*box of possible roots (x = first root, y = second root)*

**LINE 2** $x + y = [-4, 4, 14, 22]$
*vertex v means $r_1 + r_2 = 2v$*

→ Only $x + y = -4$ misses the box. So $v = -2$ is impossible: A.

Try it: put sliders on the two roots, $r_1$ between $-5$ and 3 and $r_2$ between 3 and 21, and watch the vertex move only between $-1$ and 12.

### BY HAND

1. $h(-5) > 0, h(3) < 0, h(21) > 0$: opens up, one root in $(-5, 3)$, one in $(3, 21)$.
2. Vertex $= \dfrac{r_1 + r_2}{2}$ is between $\dfrac{-5+3}{2} = -1$ and $\dfrac{3+21}{2} = 12$.
3. $-2$ is outside: A.

**Verdict:** Hand wins (about 30 s). The Desmos box is a great picture to explain it.

# 15 SOLUTION · HAND WINS

TOP 75 HARD · Q8

$$f(x) = 11x^{10} + 2x^8$$
$$g(x) = -14x^7 + 9x^5$$

The polynomial $p(x)$ is the product of the given polynomials $f(x)$ and $g(x)$. What is the coefficient of $x^{15}$ in $p(x)$?

**ANSWER** 71

**THE TRAP**

Expand the whole product, or try a regression of a degree-17 polynomial (needs 18 points and still rounds badly).

**DESMOS ROUTE**

LINE 1 $p(x) = (11x^{10} + 2x^8)(-14x^7 + 9x^5)$

LINE 2 $x_1 = [1...18]$

LINE 3 $p(x_1) \sim a_{17}x_1^{17} + \cdots + a_0$
*18 unknowns: not realistic*

→ 18 unknowns. Not worth it on test day: this is what NOT to do.

**BY HAND**

1. Exponents add: pairs that make 15 are $10 + 5$ and $8 + 7$.
2. $11 \cdot 9 = 99$ and $2 \cdot (-14) = -28$.
3. $99 - 28 = 71$.

**Verdict:** Hand wins: 10 s against several minutes.

WHERE TODAY CAME FROM

# Every question today came from two books.

Each question had a code. That code is its place in our books, along with a practice twin and the full solution, both ways.

## SAT Desmos Bible

212 SAT-style questions, and a practice twin for every one.

56 Desmos skills in 12 moves, each one tested on real questions.

Desmos, by hand, or both: every solution shows the call.

OPEN THE DESMOS BIBLE →

## Top 75 Hard

The 75 hardest Math question types, Module 2 level.

Full step-by-step solutions with the Desmos route and the hand route.

Notes on the traps students fall for.

OPEN TOP 75 HARD →

- **Session 2:** Geometry and Trigonometry, then Problem Solving and Data.
- **Session 3:** The theory you cannot miss, and your test-day plan.
- **Before Session 2:** redo every question you got wrong today, without looking at the solution.

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